

CHARACTERISTICS OF BELL VIBRATIONS.

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A bell is a metallic structure designed for the production of sound of a pleasing quality. The quality depends upon the relative amplitudes and frequencies of the series of tones of the bell, which in turn depend on its shape and thickness. Most bells are essentially a cylinder with one end closed by a flat plate and with a flared out portion at the other end. The closed end is called the crown and the open end, where the thickness is considerably increased, is called the sound bow.

Considering the bell as a modified cylinder or as a curved shell, it is to be expected that its flexural vibrations would be characterized by nodal lines, which are equidistant meridians, and by parallel circles of latitude. The crown is of course a circular node for all vibrations. For this class of vibrations the number of meridian nodes cannot be less than four, and must be even, but there can be any number of circular nodes. In this paper a vibration will be designated by two numbers; the first, its number of meridian nodes; and the second, its number of circular nodes. Thus a 6-3 has six meridian nodes and three circular nodes, excluding the node at the crown.

In addition to the above form of vibration, two other types are possible; the first, an expansion and contraction without deformation of the circular shape, having any number of circular nodes; and the second, a vibration which resembles the compressional or longitudinal vibrations of a rod with two meridian lines of no radial motion. The frequencies of all of these vibrations can be calculated for certain vibrating systems, such as a cylinder or a flat plate, but for an irregular shape like a bell this calculation is impossible.

In the sound of a bell distinct beats can often be heard in one or more of the partials. According to Lord Rayleigh this is due to lack of symmetry in the bell. The same effect can be

produced by applying a load to the bell. If the system of meridian nodes is located so that this load is at an antinode, the frequency of the tone will be slightly lower than if the load is at a node. When the bell is struck at a node for either one of these tones, the other is excited alone, but when the blow is struck at any other point, both tones are excited simultaneously and beats are heard.

This phenomenon of beats was used by Lord Rayleigh in his investigation of bell tones. In his method the bell is tapped at various points around the circumference and the tone in question is selected by the use of Helmholtz resonators. Due to a local load fastened to the bell or to its natural lack of symmetry, beats are produced. If the point of excitation is moved around the circumference these beats will disappear for a number of points equal to double the number of meridian nodes. The circular nodes are found by tapping the bell along a meridian, and noting the points at which the intensity for the tone in question is a minimum.

Another method of investigating bells is that used by A. B. Simpson, in which the stem of a tuning fork is held in contact with the bell at different points. If the frequency of the fork coincides with one of the partials of the bell, a tone will be produced, the loudness depending on the location of the point of contact. This gives a method for finding circular nodes, but for the location of meridian nodes the bell must be restrained at one or more points while the fork is moved around the circumference to determine the number of points of least sound excitation.

Professor A. T. Jones in his work on bell vibrations, has used a resonator connected by a rubber tube to a stethoscope with which he listened to the sound coming from different parts of the bell, which is always struck at the same spot.

All of these methods depend on observations made on a sound which is constantly diminishing, and are therefore somewhat uncertain. Simpson's method is limited by the necessity of using a tuning fork of the same frequency as the tone. The methods in which the bell is excited by tapping are open to objection because of the difficulty of picking out one tone from the great number produced, even with the use of a resonator. This is especially true when working with high

frequencies. Besides this difficulty, Jones' method of listening to the sound in the air is made very uncertain by the interference with the sound from other parts of the bell. This is especially confusing when working in a room where standing waves are present.

In order to increase the range of tones to be investigated, and to get more certain and easily obtained results, the excitation should be continuous, it should excite one frequency at a time, and it should not change the frequencies of the free bell tones by loading or otherwise. In regard to the detection of the motion of the bell, the best results should be obtained by using a device which responds to the motion of the bell itself, and to motion in one direction only, as the nodes for radial motion are antinodes for tangential motion.

The tone was excited in these experiments by means of a vacuum tube oscillator, the current from which passed through an ear-phone whose pole pieces rested on a small piece of rubber covered iron in contact with the bell. The frequency of the oscillator was varied until it coincided with one of the partials of the bell; this condition being indicated by a sound maximum. To make sure that the frequency thus found was the same as that excited by tapping, a tuning fork was loaded until it gave a certain number of beats per second with the oscillator tone, and then the number of beats was counted when the tone was produced by tapping. The number of beats was found to be the same except in the case of the smallest bell tested, and even here the difference was removed by supporting the excitor so that its weight was not an appreciable load on the bell.

The first instrument used in this test to observe the motion of the bell was the pickup of a phonograph connected to the ears by rubber tubes. In this device the motion of a pin is transmitted by a lever to a diaphragm which varies the air pressure in a small cavity behind it, producing sound. This was quite sensitive, but since one side of the diaphragm was open to the air, confusing phase differences between the air vibrations and those of the bell were produced, depending upon the way the diaphragm faced. Another disadvantage was, that it responded to motion of the pin in two directions. These defects were removed in an instrument devised by Mr.

B. E. Eisenhour, of the Riverbank Laboratories. In this pickup the motion of the bell in one selected direction is transmitted to an entirely enclosed diaphragm, and the sound from the cavity behind the diaphragm carried to the ears by tubes, as before.

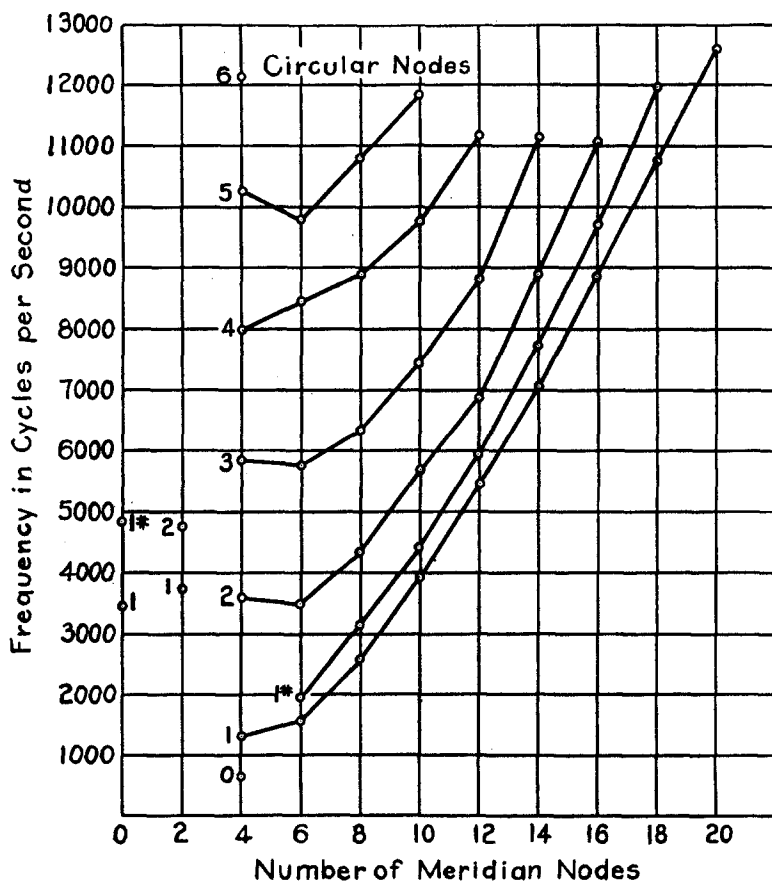
The instrument was checked for sensitivity by holding it in contact with the stem of a tuning fork, and by this means the fork could be heard even after the sound from the prongs was inaudible when held to the ear. For certain frequencies it was found that the response was cut down by resonance effects in the tubes carrying the sound to the ears. Different lengths of tubes were used for these ranges of frequencies, which were located by testing the instrument with a set of tuning forks.

The procedure of finding and classifying the vibrations was as follows: the excitor was placed in contact with the bell, and the frequency of the oscillator continuously changed until the bell was heard to respond, either directly by ear or by the use of the listening instrument. The number of meridian and circular nodes was then found by exploration over the surface of the bell with the listening device, and the frequency was determined by comparison with a set of tuning forks covering the range between 512 and 4,096 D.V. To determine frequencies higher than 4,096, a second oscillator was tuned to a known multiple of the frequency of a fork and the difference tone between this frequency and the one in question was determined by the use of the set of forks. With this arrangement it was easy to determine frequencies up to 13,000 cycles per second, which was the limit of the investigation. Frequencies below 512 were found by comparing the harmonics of the oscillator with the set of forks.

All the tones presented in the results were not discovered the first time the range of frequencies was traversed, but when the results of the first run were plotted as frequency against number of nodes, the regular series of curves obtained gave an indication of the frequencies of the missing tones. The fact that some tones were missed in the first run is due to the quite considerable differences in loudness, and to the possibility of two occurring at very nearly the same frequency. In this latter case, the two might be mistaken for one form of vibration with a slightly different position of the nodal pattern, due

to lack of symmetry in the vibrating system, as explained above. By moving the excitor around the circumference, one of the pair may be made to disappear for a number of points equal to the number of meridian nodes, thus checking the

FIG. 1.



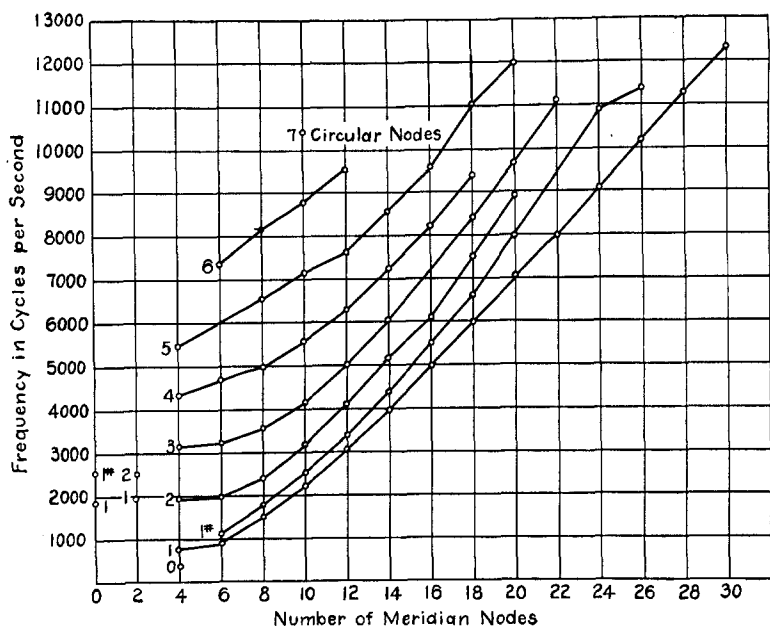
Nodal characteristics of the partials of a 35 lb. bell.

explanation of beats given by Lord Rayleigh. When the bell manufacturers pick out a certain point at which the bell is to be struck, it is in the effort to avoid beats by exciting only one of the two tones which would produce the most objectionable interference.

Three bells of different sizes were investigated, and the

tones found are presented in the form of a table. The weights of the bells were: 800, 250, and 35 lbs., and the frequencies of the hum tones were *C*-259 *F*# 367, and *E* $\flat$ 650, respectively. The most complete set of partials was found for the 35 lb. bell, which gave 44 tones below the frequency limit, 13,000 cycles per second. The frequency of the tone was plotted against the number of meridian nodes, as shown in Fig. 1, giving a curve for each number of circular nodes.

FIG. 2.



Nodal characteristics of the partials of a 250 lb. bell.

The 68 tones found in the 250 lb. bell are shown in Table 1, and are plotted in Fig. 2. Since the fundamental of this bell is nearly an octave lower than the small bell, more tones were found, involving larger numbers of circular and meridian nodes, before the frequency limit was reached, but the series of tones are not as complete. The largest number of meridian nodes was 30, while 7 circular nodes were found.

The work on the 800 lb. bell was carried only far enough to be sure that the series of tones was similar to those of the two

smaller bells. The results from this bell are also shown in Table I.

An interesting series of tones was found in which a circular node was located at the sound bow very close to the rim. To distinguish this from the tone having a circular node between the sound bow and the crown, its number of circular nodes will be designated as 1#. The motion of the bell for this 1# tone is maximum about half way between the crown and sound bow, and there is almost no motion at the sound bow. This position of the node is due to the inertia concentrated at the sound bow. An analogous vibration can be produced by holding one end of a flat spring in a vise and attaching a

TABLE I.

Tone.	Meridian Nodes.	Circular Nodes.	Frequency 250 lb. Bell.	Frequency 35 lb. Bell.	Frequency 800 lb. Bell.
1	4	0	367 D.V.	647 D.V.	269 D.V.
2	4	1	731	1,304	517
3	6	1	861	1,552	622
4	6	1#	1,093	1,968	830
5	8	1	1,468	2,596	1,034
6	8	1#	1,766	3,120	
7	0	1	1,850	3,430	1,308
8	4	2	1,915	3,588	1,404
9	6	2	1,934	3,482	1,413
10	2	1	1,952	3,734	1,487
11	10	1	2,200	3,920	1,540
12	8	2	2,384	4,304	1,796
13	10	1#	2,500	4,388	2,064
14	2	2	2,522	4,635	
15	0	1#	2,592	4,816	2,495
16	12	1	3,024	5,440	2,112
17	4	3	3,136	5,824	2,181
18	10	2	3,168	5,680	2,414
19	6	3	3,200	5,752	2,320
20	12	1#	3,368	5,926	2,830
21	8	3	3,580	6,312	2,620
22	14	1	3,936	7,078	2,752
23	12	2	4,096	6,848	3,140
24	10	3	4,160	7,424	3,111
25	4	4	4,316	7,974	
26	14	1#	4,381	7,712	3,688
27	6	4	4,690	8,432	
28	16	1	4,928	8,848	3,430
29	8	4	4,968	8,896	3,621
30	12	3	4,992	8,800	3,780
31	14	2	5,136	8,880	
32	4	5	5,450	10,272	
33	16	1#	5,472	9,696	
34	10	4	5,504	9,744	4,072
35	6	5		9,792	

TABLE I.—*Continued.*

Tone.	Meridian Nodes.	Circular Nodes.	Frequency 250 lb. Bell.	Frequency 35 lb. Bell.	Frequency 800 lb. Bell.
36	18	1	5,942	10,720	4,126
37	14	3	6,009	11,120	
38	16	2	6,016	11,072	4,672
39	12	4	6,246	11,158	
40	8	5	6,528	10,800	
41	18	1#	6,594	11,936	
42	20	1	7,024	12,592	
43	10	5	7,120	11,840	
44	4	6		12,160	
45	14	4	7,200		
46	6	6	7,332		
47	18	2	7,488		
48	12	5	7,597		
49	22	1	7,936		
50	20	1#	7,978		
51	8	6	8,120		
52	16	4	8,186		
53	18	3	8,381		
54	14	5	8,510		
55	10	6	8,736		
56	20	2	8,880		
57	24	1	9,040		
58	18	4	9,376		
59	12	6	9,504		
60	16	5	9,568		
61	20	3	9,648		
62	26	1	10,144		
63	10	7	10,400		
64	24	1#	10,830		
65	18	5	11,008		
66	22	3	11,072		
67	28	1	11,232		
68	26	2	11,332		
69	20	5	11,968		
70	30	1	12,238		

weight to the other end. When the spring is struck a sharp blow, it will vibrate transversely without moving the weight, except for a slight motion perpendicular to the motion of the spring. This system of circular nodes was found for each number of meridian nodes from six up. There seems to be no reason why a 4-1# should not occur, but it could not be found in any of the bells tested.

Two tones were found in each bell consisting of expansion and contraction without deformation of the circular shape. The number of circular nodes was 1 and 1#. This form of vibration was not found for larger numbers of circular nodes, and there was no way to tell just where to look for the more

complicated forms, as it did not follow the same sort of a curve as the vibrations having meridian nodes.

Two tones, having two meridian nodes, were found in each bell, a 2-1 and a 2-1#. The two meridians, are merely lines of no radial motion, as the radial component of the motion on either side of the line is in the same direction. On investigating the direction of the motion, it was found that it was not radial as in all the other tones, but all parts moved parallel to each other, perpendicular to a plane passed through the two meridian lines. This is different from the transverse vibration where a deformation of the circular shape would produce a minimum of four meridian nodes.

In every case except three, the frequencies increased with the number of meridian nodes for a given number of circular nodes. These exceptions occurred in the 35 lb. bell where the 4-2, 4-3 and 4-5 had higher frequencies than the 6-2, 6-3, and 6-5, respectively. These reversals can be explained by the fact that the circular nodes did not remain in the same place as the number of meridian nodes was increased. Thus the shifting of the position of a given number of circular nodes in the unequal cross-section of the bell, counteracted the frequency rise that would be expected from an increase in the number of meridian nodes. In all cases the rise of frequency as the number of nodes increased from four to six, was small.

The simplest form of vibration with no circular nodes was found only with four meridian nodes. Excluding this series, it can be seen, by referring to the curves, that the 47 partials below 12,000 D.V. found in the 35 lb. bell are complete except for one tone, the 4-1#, not found in any bell.

The results, here given, agree with those of Lord Rayleigh for the first five partials, except that he did not find any circular node for the fourth partial which is located at the sound bow. He noticed, however, that this tone was best excited by tapping the middle part of the bell. A. T. Jones found the number of meridian nodes for ten partials, and the number of circular nodes for the first seven of these. A comparison of these results is made in Table 2. It is believed that the improved apparatus used and the completeness of the series of tones found justify confidence in the results presented.

TABLE 2.

Comparison of the Lower Partial with Those Found by Other Investigators.

	Lord Rayleigh.	A. T. Jones.	This Investigation.
1	4-0	4-0	4-0
2	4-1	4-1	4-1
3	6-1	6-0	6-1
4	6-?	6-1	6-1#
5	8-1	8-1	8-1
6		8-2	8-1#
7			0-1
8			4-2
9			6-2
10			2-1
11		10-2	10-1
12		8-?	8-2
13		10-?	10-1#
14			2-2
15			0-1#
16		12-?	12-1

No attempt was made to provide a formula for the curves of frequency plotted against the number of circular nodes, as the shape of a bell is so irregular and bells made by different manufacturers do not, in general, have the same shape. Calculations of the frequency of the tones having two meridians were made by assuming that they were the same as the compressional waves in a rod with a length equal to one-half the circumference of the bell at the soundbow. For the 35 lb. and the 250 lb. bell the frequencies as calculated were 3,660 and 1,830 D.V. and as measured in the 2-1 form they were 3,734 and 1,952 D.V. respectively. This calculation does not take into account the varying diameter of the bell, the presence of a circular node, or the fact that the vibrating segment is not straight but curved.

In locating tones it was found useful to plot frequency against the number of circular nodes, as well as against the number of meridian nodes, as in the curves presented. This gave a set of curves one for each number of meridian nodes, but is not an independent way of plotting the results.

The quality of a bell is commonly regarded as being determined by the relative frequencies of the first five partials. These are supposed to be according to the ratios 1-2-2.4-3-4, or in musical terms; a fundamental or hum note, its octave, a

minor third, a perfect fifth, and the next octave. Thus the musical notes corresponding to the first five partials of a C bell are: C_1 , C_2 , E_{b2} , G_2 , C_3 . Some writers extend this to seven tones, including a major third and a perfect fifth in the next octave.

The better bell manufacturers produce the proper relationship of these five tones by design of the bell casting, and by grinding or turning the bell at different places. It is difficult to avoid some grinding, since, even when casting two bells of the same size, a slight difference in the composition of the metal or in the rate of cooling would prevent duplication. Tuning is rather a difficult process as the five tones are interrelated, and work must be done in certain regions where some tones are affected more than others. The tuning is done by the removal of metal in certain relations to the positions of the circular nodes, and is possible because they are located at different points in the bell.

It is considered difficult to make small bells with a pleasing tone. This is probably due to the difference tones between the partials, and to beats, both of which are more pronounced in the high frequency bells. Since the tones are in the ratios 1-2-2.4-3-4, the difference tones between the second, fourth, and fifth tones should be the same pitch, and if any of these are a little off pitch, beats will be produced. Low frequency difference tones also give more trouble in small bells since the difference tone between two tones above 2,500 D.V. usually seems louder than the primary tones.

There has been considerable discussion about the pitch and character of the so-called strike tone of a bell. When a bell is struck, a complex sound is produced which gives the sensation of a certain pitch. This predominating pitch is called the strike or striking tone. Professor Jones gives it as the octave below the fifth tone, and regards its origin as something of a mystery. In the bells here investigated the pitch seemed to be the same as the second partial, which was not far from being an octave below the fifth partial. In such a complex sound where the various components are dying out at different rates, it is hard to judge what tone or combination of tones predominates.

In conclusion I wish to express my appreciation of the

assistance rendered by the staff of the Riverbank Laboratories, especially to Mr. B. E. Eisenhour whose many valuable suggestions and improved listening device were very helpful in the investigation.

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